

GRADE 12

MATHEMATICS PAPER 2
MARKING GUIDELINE

QUESTION 1

MATHS LIT	25	38	40	46	47	12	49	49	54	55	59	64
ENGLISH	34	53	62	40	44	20	50	55	61	63	54	59

1.1		<p>✓ ✓ ✓ all 12 points plotted correctly</p> <p>✓ ✓ 7 – 11 points plotted correctly</p> <p>✓ 2 – 6 points plotted correctly</p>
(3)		

1.2	$a = 17,32 \quad b = 0,72$ $\hat{y} = 17,32 + 0,72x$	<p>✓ ✓ for a or b</p> <p>✓ for b or a</p> <p>✓ \hat{y}</p>
(4)		

1.3	See 1.1 above	<p>✓ for y-int</p> <p>✓ for any other point</p>
(2)		

1.4	59,06	✓ ✓ (2)
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1.5	71,82	✓ ✓ (2)
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1.6	$r = 0,808 = 0,81$	✓ ✓ (2)
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1.7	It is a strong positive relationship.	<p>✓ strong</p> <p>✓ positive (2)</p>
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1.8	$\sigma = 13.95727608... = 13.96$	✓ ✓ (2)
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Mathematics Paper 2
QUESTION 2

2.1	$M = \left(\frac{-4+9}{2}; \frac{2-2}{2} \right) = \left(\frac{5}{2}; 0 \right)$	✓ x-coord. ✓ y-coord. (2)
2.2	$\left(\frac{x+8}{2}; \frac{6+y}{2} \right) = M = \left(\frac{5}{2}; 0 \right)$ parallelogram ✓ $\frac{x+8}{2} = \frac{5}{2} \quad \therefore x = -3 \quad \checkmark \quad \frac{6+y}{2} = 0 \quad \therefore y = -6 \quad \checkmark$	(3)
2.3	$m_{BC} = \frac{y_B - y_C}{x_B - x_C} = \frac{-2+6}{9-8} = 4$	✓ for subst. ✓ answer (2)
2.4	$m_{AB} = \frac{y_A - y_B}{x_A - x_B} = \frac{6+2}{-3-9} = -\frac{2}{3}$ <p style="text-align: right;">CA from 2.2</p> $y - 6 = -\frac{2}{3}(x + 3)$ $y = -\frac{2}{3}x + 4$	✓ for subst. ✓ answer ✓ for subst. ✓ answer (4)
2.5	$m_{AD} = m_{BC}$ parallelogram ✓ CA from 2.3 $\theta = \left[180 + \overset{\checkmark}{\tan^{-1}\left(-\frac{2}{3}\right)} \right] - \overset{\checkmark}{\tan^{-1}(4)} = 70,35^\circ \quad \checkmark$	(4)
2.6	$m_{BC} \cdot m_{BD} = 4 \times \frac{-2-2}{9+4}$ $= 4 \times \frac{-4}{13}$ $= \frac{-16}{13} \neq -1$ <p>∴ ΔDBC is not right angled triangle</p>	✓ m_{BD} ✓ $\frac{-4}{13}$ ✓ statement

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QUESTION 3

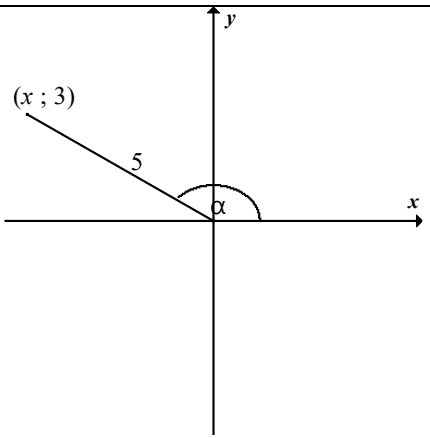
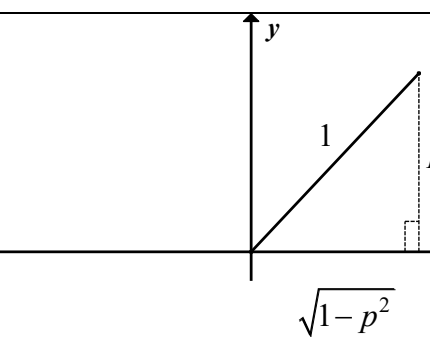
3.1	$r^2 = \left(\frac{5}{2} - 5\right)^2 + (2 - 0)^2 = \frac{41}{4}$ $\therefore \left(x - \frac{5}{2}\right)^2 + (y - 2)^2 = \frac{41}{4}$ $x^2 - 5x + \frac{25}{4} + y^2 - 4y + 4 = \frac{41}{4}$ $x^2 - 5x + y^2 - 4y = 0$	<p>✓ subst. ✓ r^2</p> <p>✓ for subst.</p> <p>✓ for expand.</p> <p>(4)</p>
3.2	$m_{BC} = \frac{y_B - y_C}{x_B - x_C} = \frac{0 - 2}{5 - \frac{5}{2}} = -\frac{4}{5}$ $\therefore m_{\tan} = \frac{5}{4}$ $y = \frac{5}{4}(x - 5)$ $y = \frac{5}{4}x + \frac{25}{4}$	<p>✓ for subst. ✓ answer</p> <p>✓ for m</p> <p>✓ for subst. ✓ answer</p> <p>(5)</p>
3.3	<p>$\angle AOB = \theta$ tan-chord thm</p> $\therefore m_{OA} = \tan \theta = \tan(78.69) = 4.999969358... = 5$ $\therefore y = 5x$	<p>✓</p> <p>✓</p> <p>✓ answer (3)</p>
3.4	<p>Subst. $y = 5x$ into $\left(x - \frac{5}{2}\right)^2 + (y - 2)^2 = \frac{41}{4}$:</p> $\therefore \left(x - \frac{5}{2}\right)^2 + (5x - 2)^2 = \frac{41}{4}$ $x^2 - 5x + \frac{25}{4} + 25x^2 - 20x + 4 = \frac{41}{4}$ $26x^2 - 25x = 0$ $x(26x - 25) = 0$ $\therefore x = 0 \text{ or } x = \frac{25}{26}$ $\therefore y = \frac{125}{26}$ $\therefore A = \left(\frac{25}{26}; \frac{125}{26}\right)$	<p>✓ for subst.</p> <p>✓ expand ✓ expand</p> <p>✓ x</p> <p>✓ y</p> <p>(5)</p>

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3.5.1	<p>Let d be the distance between the centres of the circles.</p> $d^2 = \left(\frac{5}{2} + \frac{5}{2}\right)^2 + (2-3)^2 = 26$ $r_1^2 + r_2^2 = \frac{41}{4} + \frac{61}{4} = \frac{102}{4} = 25,5 < 26$ <p>\therefore circles intersect at 2 distinct points.</p>	<p>✓ d^2</p> <p>✓ $r_1^2 + r_2^2$</p> <p>✓ for <26</p> <p>(3)</p>
3.5.2	<p>Subst. O and A into $\left(x + \frac{5}{2}\right)^2 + (y-3)^2 = \frac{61}{4}$:</p> $\left(\frac{5}{2}\right)^2 + (-3)^2 = \frac{25}{4} + 9 = \frac{61}{4} = RHS$ $\left(\frac{25}{26} + \frac{5}{2}\right)^2 + \left(\frac{125}{26} - 3\right)^2 = \left(\frac{45}{13}\right)^2 + \left(\frac{47}{26}\right)^2 = \frac{61}{4} = RHS$	<p>✓</p> <p>✓</p> <p>(2)</p>

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QUESTION 4

4.1			
4.1.1	$x^2 + y^2 = r^2$ $x^2 + 3^2 = 5^2$ $x = -\sqrt{5^2 - 3^2} = -4$ $\tan \alpha = \frac{3}{-4} = -\frac{3}{4}$		<p>✓ sketch ✓ for x ✓ tan</p> <p>(3)</p>
4.1.2	$\cos(90^\circ + \alpha) = -\sin \alpha = -\frac{3}{5}$	<p>✓ $-\sin \alpha$ ✓ answer</p> <p>(2)</p>	
4.1.3	$\cos 2\alpha = 1 - 2\sin^2 \alpha$ $= 1 - 2\left(\frac{3}{5}\right)^2 = \frac{7}{25}$	<p>✓ $1 - 2\sin^2 \alpha$ ✓ subst. ✓ answer</p> <p>(3)</p>	
4.2			
4.2.1	$x^2 + y^2 = r^2$ $x^2 + p^2 = 1$ $x = \sqrt{1 - p^2}$ $\cos(-38^\circ) = -\cos 38^\circ = \sqrt{1 - p^2}$		<p>✓ $\sqrt{1 - p^2}$ ✓ $-\cos 38^\circ$ ✓ answer</p> <p>(3)</p>
4.2.2	$\sin 76^\circ = \sin 2(38^\circ)$ $= 2 \sin 38^\circ \cos 38^\circ$ $= 2p\sqrt{1 - p^2}$	<p>✓ $2 \sin 38^\circ \cos 38^\circ$ ✓ answer</p> <p>(2)</p>	

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4.3	$\frac{\sin 150. \tan 225}{\sin(-30). \sin 420}$ $= \frac{\sin 30. \tan 45}{-\sin 30. \sin 60}$ $= -\frac{1}{\frac{\sqrt{3}}{2}}$ $= -\frac{2}{\sqrt{3}}$	<p>✓ ✓ ✓ ✓ reduction ✓ subst special angle. ✓ answer</p>
		(6)

QUESTION 5

5.1	$\frac{2 \sin^2 x}{2 \tan x - \sin 2x} = \frac{1}{\tan x}$ <p>LHS :</p> $\frac{2 \sin^2 x}{2 \tan x - \sin 2x}$ $= \frac{2 \sin^2 x}{\frac{2 \sin x}{\cos x} - 2 \sin x \cdot \cos x} \quad \checkmark A \quad \checkmark A$ $= \frac{2 \sin^2 x \cdot \cos x}{2 \sin x - 2 \sin x \cdot \cos^2 x} \quad \checkmark CA$ $= \frac{2 \sin^2 x \cdot \cos x}{2 \sin x (1 - \cos^2 x)} \quad \checkmark CA$ $= \frac{2 \sin^2 x \cdot \cos x}{2 \sin x \cdot \sin^2 x} \quad \checkmark CA$ $= \frac{\cos x}{\sin x} \quad \checkmark CA$ $= \frac{1}{\tan x} = \text{RHS}$	<p>1 identity of tan x 1 identity of sin 2x</p> <p>1 multiplying by cos x</p> <p>factorising denominator</p> <p>use of Pythagorean identity</p> <p>simplification</p>
		(6)

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5.2	$\frac{\sin 3x + \sin x}{\cos 2x + 1}$ $= \frac{\sin(2x+x) + \sin x}{2\cos^2 x - 1 + 1}$ $= \frac{\sin 2x \cos x + \cos 2x \sin x + \sin x}{2\cos^2 x}$ $= \frac{2\sin x \cos x \cos x + \cos 2x \sin x + \sin x}{2\cos^2 x}$ $= \frac{\sin x(2\cos^2 x + \cos 2x + 1)}{2\cos^2 x}$ $= \frac{\sin x(2\cos^2 x + 2\cos^2 x - 1 + 1)}{2\cos^2 x}$ $= 2\sin x$	<p>✓ $3x = (2x + x)$</p> <p>✓ double angle of $\cos 2x$</p> <p>✓ expansion</p> <p>✓ $\sin 2x = 2\sin x \cos x$</p> <p>✓ common factor</p> <p>✓ answer (6)</p>
		[12]

QUESTION 6

6.1.1	$x = -300$ or $x = 60$	✓ each (2)
6.1.2	$x = 60 + 360k, k \in Z$	<p>✓ 60</p> <p>✓ $360k$</p> <p>✓ $k \in Z$ (3)</p>
6.2.1	$x = -210^\circ$ or -30°	✓ each (2)
6.2.2	$-360 \leq x < -210$ or $30 \leq x < 150$	<p>✓ for each</p> <p>CORRECT</p> <p>interval (2)</p>

[9]

QUESTION 7

7.1	$\begin{aligned} \mathbf{MQ}^2 &= k^2 + (2k)^2 - 2k(2k)(\cos 2\theta) \\ &= k^2 + 4k^2 - 4k^2(1 - 2\sin^2 \theta) \\ &= 5k^2 - 4k^2 + 8k^2 \sin^2 \theta \\ &= k^2 + 8k^2 \sin^2 \theta \\ &= k^2(1 + 8\sin^2 \theta) \\ \mathbf{MQ} &= k\sqrt{1 + 8\sin^2 \theta} \end{aligned}$	<ul style="list-style-type: none"> ✓ subst.in cos rule ✓ double angle formula ✓ expansion. ✓ for simplification . ✓ for factorisation . <p style="text-align: right;">(5)</p>
7.2	$\begin{aligned} \mathbf{MQ} &= 139,5m\sqrt{1 + 8\sin^2 42^\circ} \\ &= 299 m \end{aligned}$	<ul style="list-style-type: none"> ✓ subst. ✓ answer <p style="text-align: right;">(2)</p>
		[7]

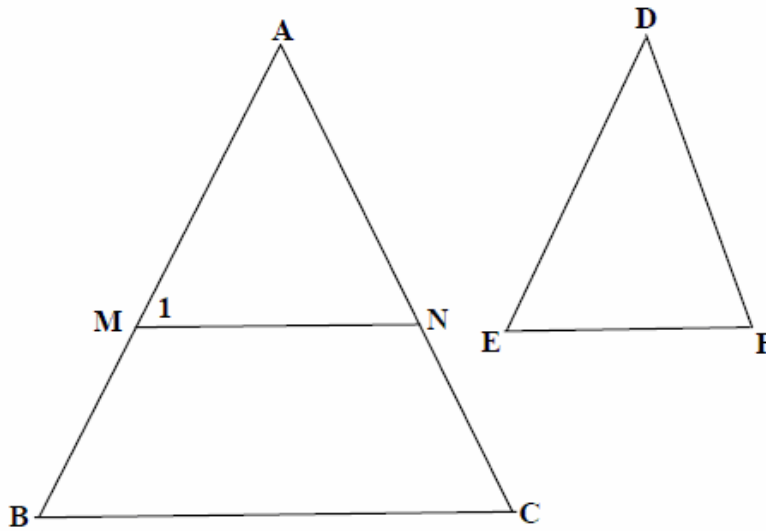
QUESTION 8

8.1.1	Equal to the angle subtended by the same chord in the alternate segment	✓ answer
8.1.2	supplementary	✓ answer
8.2.1	$\hat{M}_1 = 2 \hat{R}_2$ $= 56^\circ$ $\hat{S}_2 = 56^\circ$	✓ \angle at centre ✓ answer ✓ answer alt. angles
8.2.2	$PM = SM$ $\therefore \hat{P}_1 + \hat{P}_2 = \hat{S}_1$ $= 62^\circ$ $\therefore \hat{PSR} = 118^\circ$	✓ equal radii ✓ answer ✓ answer
8.2.3	$\hat{Q} = 62^\circ$	✓ answer/angle ✓ Opp angles of cyclic quad
8.2.4	$\hat{P}_2 + \hat{P}_3 = \hat{Q} = 62^\circ$ $\hat{P}_2 = 28^\circ$ $\therefore \hat{P}_3 = 34^\circ$	✓ QR=PR ✓ alt angles ✓ answer

QUESTION 9		
9.1	$\hat{S} = \hat{Q}_2 = x$ (alt angles TQ PS) $\hat{S} = \hat{W}_1 = x$ (\angle s in same segment) $\hat{S} = \hat{Q}_4 = x$ (\angle btwn tang and chord)	✓ statement & reason ✓ statement ✓ Reason ✓ statement ✓ Reason
9.2.1	$\hat{R}_1 = \hat{S} + \hat{Q}_3$ ext \angle Δ ✓ S/R $\hat{L}_3 = \hat{W}_1 + \hat{Q}_1$ ext \angle Δ ✓ S/R but $\hat{S} = \hat{V}$ 8.2.1 n 8.1 ✓ S $\hat{Q}_1 = \hat{Q}_3$ ✓ S \angle s subt by equal chords ✓ R $\therefore \hat{R}_1 = \hat{L}_3$ (5)	
9.2.2	$\hat{W}_1 = \hat{Q}_4$ ✓ S \angle between tang and chord ✓ R $\hat{Q}_1 = \hat{Q}_3$ \angle s subt by equal chords ✓ S/R $\therefore \hat{Q}_4 + \hat{Q}_3 = \hat{W}_1 + \hat{Q}_1$ but $\hat{L}_3 = \hat{W}_1 + \hat{Q}_1$ ext. \angle of Δ ✓ S/R $= \hat{R}_3$ corr. \angle s equal lines parallel $\therefore \hat{R}_3 = \hat{Q}_4 + \hat{Q}_3$ Quad PRKQ is cyclic ext. \angle = int opp \angle ✓ R OR $\hat{Q}_1 + \hat{Q}_2 = \hat{Q}_3 + \hat{S}$ from 8.1 and 8.2. OR $\hat{W}_1 = \hat{Q}_4$ ✓ S \angle between tang and chord ✓ R $\hat{Q}_1 = \hat{Q}_3$ \angle s subt by equal chords ✓ S/R $\therefore \hat{Q}_4 + \hat{Q}_3 = \hat{W}_1 + \hat{Q}_1$	

QUESTION 10

10.1



Construction : On AB draw $AM = DE$ and on AC draw $AN = DF$ ✓ CONSTR

Proof : $\triangle AMN \equiv \triangle DEF$ ✓ s, \angle , s ✓ S/R

$$\begin{aligned} \therefore \hat{M}_1 &= \hat{E} && \text{Congruency} \\ &= \hat{B} && \checkmark s \end{aligned}$$

$$\therefore MN \parallel BC \quad \text{corr. } \angle s = \checkmark \text{ S/R}$$

$$\therefore \frac{AB}{AM} = \frac{AC}{AN} \quad \checkmark s \quad \text{line } \parallel \text{ to one side of } \triangle \quad \checkmark R$$

$$\therefore \frac{AB}{DE} = \frac{AC}{DF} \quad AM = DE \text{ and } AN = DF \quad \checkmark \text{ S/R}$$

(6)

10.2.1 $\frac{CE}{ED} = \frac{CT}{TA} = \frac{1}{2}$ ✓ line divides sides in prop. ✓

10.2.2 From 10.1 $\frac{CE}{ED} = \frac{1}{2}$

But $DC = 9 \text{ cm}$
 $\therefore DE = 6 \text{ cm}$
 $= BD.$
 $\therefore D$ is the midpoint of $BE.$

✓ use of ratio

✓ $DE = 6 \text{ cm}$

<p>10.2.3</p>	$\frac{FD}{TE} = \frac{BD}{BE}$ $\frac{2}{TE} = \frac{6}{12}$ <p>(TE = 24 TE = 4 cm</p> <p>ALTERNATIVE</p> <p>D is the midpoint of BE. (from 9.2.2) Then F is the midpoint of BT. ... (sides in proportion)</p> <p>∴ TE = 2FD (midpoint theorem) = 4 cm</p>	<p>✓ proportion</p> <p>✓ answer</p> <p>✓ proportion</p> <p>✓ answer</p>
<p>10.2.4</p>	$\frac{\Delta TEC}{\Delta ABC} = \frac{\Delta TEC}{\Delta TBC} \times \frac{\Delta TBC}{\Delta ABC}$ $= \left(\frac{1}{5}\right)\left(\frac{1}{3}\right)$ $= \frac{1}{15}$ <p>OR</p> $\frac{\text{area } \Delta TEC}{\text{area } \Delta ABC} = \frac{\frac{1}{2} \cdot TC \cdot EC \cdot \sin \hat{C}}{\frac{1}{2} \cdot AC \cdot BC \cdot \sin \hat{C}}$ $= \frac{TC \cdot EC}{AC \cdot BC}$ $= \left(\frac{1}{5}\right)\left(\frac{1}{3}\right)$ $= \frac{1}{15}$	<p>✓ ratios</p> <p>✓ substitution</p> <p>✓ answer</p> <p>✓ ratios</p> <p>✓ substitution</p> <p>✓ answer</p> <p>Answer Only : 3/3</p>

[15]